Low Freeze-out Temperature and High Collective Velocities in Relativistic Heavy-Ion Collisions

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Abstract

On the basis of a nine-parameter expanding source model that includes special relativity, quantum statistics, resonance decays, and freeze-out on a realistic hypersurface in spacetime, we analyze in detail invariant π^+, π^-, K^+, and K^- one-particle multiplicity distributions and π^+ and K^+ two-particle correlations in nearly central collisions of Si + Au at $p_{\rm lab}/A = 14.6$ GeV/c. By considering separately the one-particle data and the correlation data, we find that the central baryon density, nuclear temperature, transverse collective velocity, longitudinal collective velocity, and source velocity are determined primarily by one-particle multiplicity distributions and that the transverse radius, longitudinal proper time, width in proper time, and pion incoherence fraction are determined primarily by two-particle correlations. By considering separately the pion data and the kaon data, we find that although the pion freeze-out occurs somewhat later than the kaon freeze-out, the 99% confidence-level error bars associated with the two freeze-outs overlap. By constraining the transverse freeze-out to the same source time for all points with the same longitudinal position and by allowing a more flexible freeze-out in the longitudinal direction, we find that the precise shape of the freeze-out hypersurface is relatively unimportant. By regarding the pion and kaon one-particle data to be unnormalized, we find that the nuclear temperature increases slightly, but that its uncertainty increases substantially. By including proton one-particle data (which are contaminated by spectator protons), we find that the nuclear temperature increases slightly. These detailed studies confirm our earlier conclusion based on the simultaneous consideration of the pion and kaon one-particle and correlation data that the freeze-out temperature is less than 100 MeV and that both the longitudinal and transverse collective velocities—which are anti-correlated with the temperature—are substantial. We also discuss the flaws in several previous analyses that yielded a much higher freeze-out temperature of approximately 140 MeV for both this reaction and other reactions involving heavier projectiles and/or higher bombarding energies.

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I. INTRODUCTION

Experimentalists around the world are vigorously searching for the quark-gluon plasma—a predicted new phase of nuclear matter where quarks roam almost freely throughout the medium instead of being confined to individual nucleons [1–5]. Such a plasma is believed to have existed in the first 10 μs of the universe during the big bang and could be produced in the laboratory during the little bang in a relativistic heavy-ion collision.

When nuclei collide head-on at relativistic speeds, the nuclear matter is initially compressed and excited from normal nuclear density and zero temperature to some maximum values—during which pions, kaons, and other particles are produced—and then expands, with a decrease in density and temperature. The early stages of the process are often treated in terms of nuclear fluid dynamics, but at some late stage the expanding matter freezes out into a collection of noninteracting hadrons.

Measurements of the invariant one-particle multiplicity distributions and two-particle correlations for the pions, kaons, and other particles that are produced sample the density, temperature, collective velocity, size, and other properties of the system during this freeze-out. The use of two-particle correlations to extract size information was pioneered in the 1950s by Brown[1] and Twiss [6], who used two-photon correlations to measure the size of stars, and by Goldhaber *et al.* [7], who used two-pion correlations to measure the size of the interaction region in antiproton annihilation. The hope is that a sharp discontinuity in the value of one or more of the extracted freeze-out properties as a function of bombarding energy and/or size of the colliding nuclei could signal the formation of a quark-gluon plasma or other new physics.

For the extraction of the freeze-out properties from experimental measurements of invariant one-particle multiplicity distributions and two-particle correlations, a nine-parameter expanding source model that includes special relativity, quantum statistics, resonance decays, and freeze-out on a realistic hypersurface in spacetime was developed in Refs. [8,9]. The application of this model to central collisions of Si + Au at $p_{\mathrm{lab}}/A = 14.6$ GeV/c [10,11] led to the conclusion that the freeze-out temperature is less than 100 MeV and that both the longitudinal and transverse collective velocities—which are anti-correlated with the temperature—are substantial. Similar conclusions concerning a low freeze-out temperature have also been reached in Refs. [12,13]. However, other analyses [14–18] have yielded a much higher freeze-out temperature of approximately 140 MeV for both this reaction and other reactions involving heavier projectiles and/or higher bombarding energies.

Because of the importance of resolving this significant discrepancy, we perform here a more detailed analysis of exactly the same data [10,11] that were considered in Refs. [8,9].

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[1] Like many individuals throughout history, R. H. Brown preferred his middle name to his first name. Therefore, instead of using Robert H. Brown as author on his many articles and books, he used R. Hanbury Brown instead. This practice inevitably led to the citation of Brown as Hanbury Brown and even Hanbury-Brown. Although this error is widespread throughout the relativistic heavy-ion community, he is correctly listed under the last name Brown in most astronomy texts, biographical reference books, and encyclopedias.

After briefly reviewing the source model in Sec. II and the results of a simultaneous consideration of the one-particle data and correlation data in Sec. III A, we examine various subsets of the data and determine the effect of alternative assumptions on our results. In particular, we consider separately the one-particle data and the correlation data in Sec. III B and consider separately the pion data and the kaon data in Sec. III C. In Sec. III D we constrain the transverse freeze-out to the same source time for all points with the same longitudinal position and allow a more generalized freeze-out in the longitudinal direction, and in Sec. III E we regard the pion and kaon one-particle data to be unnormalized and also include proton one-particle data (which are contaminated by spectator protons). These detailed studies provide a useful background for our discussion in Sec. IV of the flaws in several previous analyses that led to an anomalously high freeze-out temperature. Our summary and conclusion are given in Sec. V.

II. NINE-PARAMETER EXPANDING SOURCE MODEL

The expanding source model introduced in Refs. [8,9] describes invariant one-particle multiplicity distributions and two-particle correlations in nearly central relativistic heavy-ion collisions in terms of nine parameters, which are necessary and sufficient to characterize the gross properties of the source during its freeze-out from a nuclear fluid into a collection of noninteracting, free-streaming hadrons. The values of these nine parameters, along with their uncertainties at 99% confidence limits, are determined by minimizing χ^2 for the types of data considered. Several additional physically relevant quantities, along with their uncertainties at 99% confidence limits, can then be directly calculated. The nine independent source freeze-out properties that we consider here are the central baryon density n, nuclear temperature T, transverse collective velocity $v_{\rm t}$, longitudinal collective velocity v_ℓ, source velocity $v_{\rm s}$, transverse radius $R_{\rm t}$, longitudinal proper time $\tau_{\rm f}$, width in proper time $\Delta\tau$, and pion incoherence fraction λ_π.

For a particular type of particle, the invariant one-particle multiplicity distribution and two-particle correlation function are calculated in terms of a Wigner distribution function, which is the phase-space density on the freeze-out hypersurface, giving the probability of producing a particle at spacetime point x with four-momentum p. It includes both a direct term [19] and a term corresponding to 10 resonance decays [20], namely the decay of meson resonances with masses below 900 MeV and strongly decaying baryon resonances with masses below 1410 MeV.

The direct part of the Wigner distribution function for a particular type of particle is given by [9,19]

$$S_{\rm dir}(x,p) = \frac{2J+1}{(2\pi)^3} \frac{p\cdot n(x)}{\exp\{[p\cdot v(x) - \mu(x)]/T(x)\} \mp 1} \ , \tag{1}$$

with the minus sign applying to bosons and the plus sign to fermions. The quantity J is the spin of the particle, $v(x)$ is the collective four-velocity, $T(x)$ is the nuclear temperature, and $\mu(x)$ is the chemical potential for this type of particle. The four-vector $n(x)$, with components

$$n_\mu(x) = \int_\Sigma d^3\sigma_\mu(x')\, \delta^{(4)}(x - x') \ , \tag{2}$$

gives the normal-pointing freeze-out hypersurface elements. The subscript Σ on the integral denotes the limits to the hypersurface for a finite-sized system. Because we are considering nearly central collisions, we assume axial symmetry and work in cylindrical coordinates in the source frame, with longitudinal distance denoted by z, transverse distance denoted by ρ, and time denoted by t. Throughout the paper we use units in which $\hbar = c = k = 1$, where $\hbar$ is Planck's constant divided by 2π, c is the speed of light, and k is the Boltzmann constant. However, for clarity, we reinsert c in the units of quantities whose values are given in the text or tables.

Integration of Eq. (1) over spacetime leads to the Cooper-Frye formula for the direct contribution to the invariant one-particle multiplicity distribution [21], namely

$$P_{\rm dir}(p) = E\,\frac{d^3N_{\rm dir}}{dp^3} = \frac{1}{2\pi m_{\rm t}}\,\frac{d^2N_{\rm dir}}{dy\,dm_{\rm t}} = \frac{2J+1}{(2\pi)^3}\int_\Sigma d^3\sigma_\mu\,\frac{p^\mu}{\exp\{[p\cdot v(x) - \mu(x)]/T(x)\} \mp 1} \ , \tag{3}$$

where E denotes the particle's energy, $m_t = \sqrt{m^2 + p_t{}^2}$ its transverse mass, and y its rapidity. The quantity m is the particle's rest mass, and $p_t = \sqrt{p_x{}^2 + p_y{}^2}$ is its transverse momentum. We assume that the source is boost invariant within the limited region between its two ends [22,23], and that it starts expanding from an infinitesimally thin disk at time $t = 0$. The transverse velocity at any point on the freeze-out hypersurface is assumed to be linear in the transverse coordinate ρ.

For a particular type of particle, the two-particle correlation function is given by [9,24,25]

$$C(K,q) = \frac{P_2(p_1,p_2)}{P(p_1)\,P(p_2)} = 1 \pm \lambda\,\frac{|\int d^4x\,S(x,K)\,\exp(iq\cdot x)|^2}{[\int d^4x\,S(x,p_1)][\int d^4x\,S(x,p_2)]} \ , \tag{4}$$

where $K = \frac{1}{2}(p_1 + p_2)$ is one-half the pair four-momentum and $q = p_1 - p_2$ is the pair four-momentum difference. The plus sign applies to bosons and the minus sign to fermions, and the quantity λ specifies the fraction of particles of this type that are produced incoherently.

The freeze-out hypersurface is specified by

$${\tau_{\rm f}}^2 = \frac{t^2 - z^2}{1 + \alpha_{\rm t}(\rho/R_{\rm t})^2} \ , \tag{5}$$

where $\tau_{\rm f}$ is the constant proper time at which freeze-out is assumed to occur along the symmetry axis of the source and $R_{\rm t}$ is the transverse radius of the source at the beginning of freeze-out. The transverse freeze-out coefficient $\alpha_{\rm t}$ specifies the radial behavior of the freeze-out and is related to the transverse velocity $v_{\rm t}$ and width $\Delta\tau$ in proper time during which freeze-out occurs. We obtain for this relationship

$$\Delta\tau = \tau_{\rm f}\left[1 - \sqrt{(1+\alpha_{\rm t})(1 - {v_{\rm t}}^2)}\right] \tag{6}$$

under the additional assumption that the exterior matter at $z = 0$ that freezes out first has been moving with constant transverse velocity $v_{\rm t}$ from time $t = 0$ until the beginning of freeze-out.

III. DETAILED ANALYSIS OF THE REACTION Si + Au AT $p_{\mathrm{lab}}/A = 14.6$ GeV/c

We now use the nine-parameter expanding source model described in Sec. II to perform a detailed analysis of nearly central collisions in the reaction Si + Au at $p_{\mathrm{lab}}/A = 14.6$ GeV/c, for which excellent experimental data were collected in Experiment E-802 [10,11] at the Alternating Gradient Synchrotron of the Brookhaven National Laboratory. We first describe the results of a simultaneous consideration of the pion and kaon one-particle and correlation data and then examine various subsets of the data and determine the effect of alternative assumptions on our results.

A. Simultaneous consideration of pion and kaon one-particle and correlation data

In Experiment E-802, invariant π^+, π^-, K^+, and K^- one-particle multiplicity distributions [10] and π^+ and K^+ two-particle correlations [11] were measured for the central 7% of collisions in the reaction Si + Au at $p_{\mathrm{lab}}/A = 14.6$ GeV/c. The nine adjustable parameters of our expanding source model have been determined by minimizing χ^2 with a total of 1416 data points for the six types of data considered, so the number of degrees of freedom ν is 1407. The error for each point is calculated as the square root of the sum of the squares of its statistical error and its systematic error, with a systematic error of 15% for π^+, π^-, and K^+ one-particle multiplicity distributions, 20% for the K^- one-particle multiplicity distribution, and zero for π^+ and K^+ two-particle correlations [10,11]. The resulting value of χ^2 is 1484.6, which corresponds to an acceptable value of $\chi^2/\nu = 1.055$. The values of nine independent freeze-out properties determined this way, along with their uncertainties at 99% confidence limits on all quantities considered jointly, are given in Table I. These values and their uncertainties were obtained earlier in Ref. [9]. The quantity n_0 appearing in Table I and in the remaining tables denotes normal nuclear density, whose value is calculated from the nuclear radius constant r_0 [26] by means of $n_0 = 3/(4\pi r_0{}^3) = 3/[4\pi(1.16\ \mathrm{fm})^3] = 0.153\ \mathrm{fm}^{-3}$.

TABLE I. Nine independent source freeze-out properties for central collisions of Si + Au at $p_{\mathrm{lab}}/A = 14.6$ GeV/c resulting from the simultaneous consideration of the pion and kaon one-particle and correlation data. The value used for normal nuclear density n_0 is 0.153 fm^{-3}.

Property	Value and uncertainty at 99% confidence
Central baryon density n/n_0	$0.145\ ^{+0.063}_{-0.045}$
Nuclear temperature T (MeV)	92.9 ± 4.4
Transverse collective velocity v_{t} (c)	0.683 ± 0.048
Longitudinal collective velocity v_ℓ (c)	$0.900\ ^{+0.023}_{-0.029}$
Source velocity v_{s} (c)	$0.875\ ^{+0.015}_{-0.016}$
Transverse radius R_{t} (fm)	8.0 ± 1.6
Longitudinal proper time τ_{f} (fm/c)	8.2 ± 2.2
Width in proper time $\Delta\tau$ (fm/c)	$5.9\ ^{+4.4}_{-2.6}$
Pion incoherence fraction λ_π	0.65 ± 0.11

B. Separate consideration of one-particle data and correlation data

We next use our expanding source model to analyze the invariant π^+, π^-, K^+, and K^- one-particle multiplicity distributions alone, for which there are a total of 656 data points. Because the pion incoherence fraction λ_π does not enter in the expression for one-particle multiplicity distributions, there are only eight parameters in this case, so the number of degrees of freedom ν is 648. The resulting value of χ^2 is 692.5, which corresponds to a value of $\chi^2/\nu = 1.069$. The values of eight independent freeze-out properties determined this way, along with their uncertainties at 99% confidence limits on all quantities considered jointly, are given in the second column of Table II. It is seen that one-particle multiplicity distributions alone determine the freeze-out central baryon density, nuclear temperature, transverse collective velocity, longitudinal collective velocity, and source velocity moderately well, but contain almost no information concerning the transverse radius, longitudinal proper time, and width in proper time.

TABLE II. Effect on nine independent source freeze-out properties for central collisions of Si + Au at $p_{\mathrm{lab}}/A = 14.6$ GeV/c of considering separately the one-particle data and the correlation data. The symbols denoting the freeze-out properties are defined in Table I.

	Value and uncertainty at 99% confidence	
Property	One-particle data	Correlation data
n/n_0	$0.145\,^{+0.079}_{-0.057}$	$1.1\,^{+2.5}_{-1.1}$
T (MeV)	92.9 ± 4.7	$70\,^{+89}_{-70}$
v_{t} (c)	0.700 ± 0.095	$0.95\,^{+0.05}_{-0.36}$
v_ℓ (c)	$0.904\,^{+0.049}_{-0.094}$	$0.92\,^{+0.08}_{-0.92}$
v_{s} (c)	$0.872\,^{+0.015}_{-0.017}$	$0.96\,^{+0.02}_{-0.12}$
R_{t} (fm)	$9\,^{+20}_{-9}$	9.6 ± 4.9
τ_{f} (fm/c)	$7\,^{+23}_{-7}$	9.6 ± 7.2
$\Delta\tau$ (fm/c)	$3.9\,^{+2.4}_{-3.9}$	$8.9\,^{+4.3}_{-6.5}$
λ_π	——	0.75 ± 0.18

In an analogous study, we use our expanding source model to analyze the π^+ and K^+ two-particle correlations alone, for which there are a total of 760 data points, or 751 degrees of freedom ν. The resulting value of χ^2 is 756.0, which corresponds to a highly acceptable value of $\chi^2/\nu = 1.007$. The values of nine independent freeze-out properties determined this way, along with their uncertainties at 99% confidence limits on all quantities considered jointly, are given in the third column of Table II. It is seen that two-particle correlations alone determine the transverse radius, longitudinal proper time, width in proper time, and pion incoherence fraction fairly well, but contain almost no information concerning the central baryon density, nuclear temperature, transverse collective velocity, longitudinal collective velocity, and source velocity.

C. Separate consideration of pion data and kaon data

In our next study, we use our expanding source model to analyze only the data for pions, namely the invariant π^+ and π^- one-particle multiplicity distributions and the π^+ two-particle correlations. For this case, there are a total of 934 data points, or 925 degrees of freedom ν. The resulting value of χ^2 is 959.9, which corresponds to an acceptable value of $\chi^2/\nu = 1.038$. The values of nine independent freeze-out properties determined this way, along with their uncertainties at 99% confidence limits on all quantities considered jointly, are given in the second column of Table III. It is seen that when pions alone are considered, the freeze-out occurs somewhat later and at a lower temperature than when both pions and kaons are considered (Table I). However, the 99% confidence-level error bars associated with the two freeze-outs overlap.

TABLE III. Effect on nine independent source freeze-out properties for central collisions of Si + Au at $p_{\rm lab}/A = 14.6$ GeV/c of considering separately the pion data and the kaon data. The symbols denoting the freeze-out properties are defined in Table I.

	Value and uncertainty at 99% confidence	
Property	Pion data	Kaon data
n/n_0	$0.01\ ^{+0.59}_{-0.01}$	$0.26\ ^{+0.17}_{-0.12}$
T (MeV)	78 ± 20	101 ± 10
$v_{\rm t}$ (c)	0.77 ± 0.14	0.62 ± 0.14
v_ℓ (c)	$0.920\ ^{+0.033}_{-0.056}$	$0.89\ ^{+0.08}_{-0.20}$
$v_{\rm s}$ (c)	$0.880\ ^{+0.014}_{-0.015}$	$0.847\ ^{+0.052}_{-0.076}$
$R_{\rm t}$ (fm)	9.5 ± 2.7	7.1 ± 2.5
$\tau_{\rm f}$ (fm/c)	11.1 ± 5.2	6.6 ± 3.8
$\Delta\tau$ (fm/c)	$8.4\ ^{+7.4}_{-4.6}$	$2.9\ ^{+7.4}_{-2.9}$
λ_π	0.75 ± 0.16	——

In an analogous study, we use our expanding source model to analyze only the data for kaons, namely the invariant K^+ and K^- one-particle multiplicity distributions and the K^+ two-particle correlations, for which there are a total of 482 data points. Because the pion incoherence fraction λ_π does not enter when kaons alone are considered, there are only eight parameters in this case, so the number of degrees of freedom ν is 474. The resulting value of χ^2 is 441.5, which corresponds to a highly acceptable value of $\chi^2/\nu = 0.931$. The values of eight independent freeze-out properties determined this way, along with their uncertainties at 99% confidence limits on all quantities considered jointly, are given in the third column of Table III. It is seen that when kaons alone are considered, the freeze-out occurs somewhat earlier and at a higher temperature than when both pions and kaons are considered (Table I). However, the 99% confidence-level error bars associated with these two freeze-outs, as well as with the separate pion freeze-out and kaon freeze-out, overlap.

D. Constraining and generalizing the freeze-out hypersurface

We next consider the effect of constraining the transverse freeze-out to the same source time for all points with the same longitudinal position, which is accomplished by holding the transverse freeze-out coefficient $\alpha_{\rm t}$ appearing in Eq. (5) fixed at zero. We once again use all 1416 data points for our six types of pion and kaon one-particle and correlation data. Because $\alpha_{\rm t}$ is held fixed, there are only eight parameters in this case, so the number of degrees of freedom ν is 1408. The resulting value of χ^2 is 1519.4, which corresponds to a value of $\chi^2/\nu = 1.079$. The values of nine independent freeze-out properties determined this way, along with their uncertainties at 99% confidence limits, are given in the second column of Table IV. Because of the strong dependence of the width in proper time $\Delta\tau$ upon the transverse freeze-out coefficient $\alpha_{\rm t}$, which is held fixed at zero, the value of $\Delta\tau$ and its uncertainty are anomalously small in this case. We see that constraining the freeze-out hypersurface in this way affects the remaining eight quantities only to within their 99% confidence-level error bars.

TABLE IV. Effect on nine independent source freeze-out properties for central collisions of Si + Au at $p_{\rm lab}/A = 14.6$ GeV/c of constraining the transverse freeze-out to the same source time for all points with the same longitudinal position (transverse freeze-out coefficient $\alpha_{\rm t}$ fixed at zero) and of allowing a more flexible freeze-out in the longitudinal direction (longitudinal freeze-out coefficient α_ℓ fixed at 0.20 c^{-2}). The symbols denoting the freeze-out properties are defined in Table I.

	Value and uncertainty at 99% confidence	
Property	$\alpha_{\rm t}$ fixed at zero	α_ℓ fixed at 0.20 c^{-2}
n/n_0	$0.139\ ^{+0.058}_{-0.042}$	$0.141\ ^{+0.061}_{-0.043}$
T (MeV)	92.6 ± 4.2	92.5 ± 4.4
$v_{\rm t}$ (c)	0.721 ± 0.033	0.663 ± 0.049
v_ℓ (c)	$0.903\ ^{+0.022}_{-0.027}$	$0.925\ ^{+0.025}_{-0.037}$
$v_{\rm s}$ (c)	$0.875\ ^{+0.014}_{-0.016}$	$0.873\ ^{+0.014}_{-0.016}$
$R_{\rm t}$ (fm)	9.5 ± 1.2	7.7 ± 1.5
$\tau_{\rm f}$ (fm/c)	6.3 ± 1.4	9.7 ± 2.2
$\Delta\tau$ (fm/c)	$1.94\ ^{+0.52}_{-0.47}$	$7.3\ ^{+4.5}_{-2.7}$
λ_π	0.64 ± 0.11	0.57 ± 0.10

In a related study, we consider the effect of allowing a more flexible freeze-out in the longitudinal direction. To accomplish this, we generalize Eq. (5) to

$$ {\tau_{\rm f}}^2 = \frac{t^2 - \alpha_\ell z^2}{1 + \alpha_{\rm t}(\rho/R_{\rm t})^2} \ , \tag{7} $$

where the longitudinal freeze-out coefficient α_ℓ allows the freeze-out along the symmetry axis of the source to occur with a dependence on longitudinal distance z that is different from that corresponding to a constant proper time (so that $\tau_{\rm f}$ no longer has this physical interpretation). By minimizing χ^2 with a total of 1416 data points for the six types of data

considered, we determined that the minimum in χ^2 occurs at a value of the longitudinal freeze-out coefficient $\alpha_\ell = 0.20\ c^{-2}$. The resulting freeze-out hypersurface is similar to that obtained in nuclear fluid-dynamical calculations by Schlei [27], which was the original motivation for this generalization. We then held α_ℓ fixed at $0.20\ c^{-2}$, so that once again there are nine parameters and 1407 degrees of freedom. The resulting value of χ^2 is 1468.5, which corresponds to an acceptable value of $\chi^2/\nu = 1.044$. The values of nine independent freeze-out properties determined this way, along with their uncertainties at 99% confidence limits, are given in the third column of Table IV. It is seen that the primary effect of this generalized freeze-out hypersurface is to increase somewhat the values of $\tau_{\rm f}$ and $\Delta\tau$, but that the precise shape of the freeze-out hypersurface remains relatively unimportant.

E. Use of unnormalized one-particle data and inclusion of proton one-particle data

Because many analyses have been performed with unnormalized one-particle data [14–18], we now consider the effect of regarding the pion and kaon one-particle data to be unnormalized. Once again, we use all 1416 data points for our six types of pion and kaon one-particle and correlation data. Because the pion and kaon one-particle normalization constants are allowed to vary, there are 11 parameters in this case, so the number of degrees of freedom ν is 1405. The resulting value of χ^2 is 1483.2, which corresponds to a value of $\chi^2/\nu = 1.056$. The values of nine independent freeze-out properties determined this way, along with their uncertainties at 99% confidence limits, are given in the second column of Table V. It is seen that the extracted temperature increases slightly, but that its uncertainty increases substantially.

TABLE V. Effect on nine independent source freeze-out properties for central collisions of Si + Au at $p_{\rm lab}/A = 14.6$ GeV/c of regarding the pion and kaon one-particle data to be unnormalized and of including proton one-particle data (which are contaminated by spectator protons). The symbols denoting the freeze-out properties are defined in Table I.

	Value and uncertainty at 99% confidence	
Property	Unnormalized one-particle data	Proton one-particle data included
n/n_0	$0.19\ ^{+0.34}_{-0.18}$	$0.140\ ^{+0.039}_{-0.034}$
T (MeV)	98 ± 27	97.0 ± 4.0
$v_{\rm t}$ (c)	0.65 ± 0.19	0.614 ± 0.033
v_ℓ (c)	$0.893\ ^{+0.041}_{-0.063}$	$0.929\ ^{+0.016}_{-0.021}$
$v_{\rm s}$ (c)	$0.875\ ^{+0.016}_{-0.018}$	$0.832\ ^{+0.020}_{-0.022}$
$R_{\rm t}$ (fm)	7.8 ± 2.4	7.4 ± 1.4
$\tau_{\rm f}$ (fm/c)	8.1 ± 2.7	8.5 ± 2.0
$\Delta\tau$ (fm/c)	$6.1\ ^{+4.7}_{-3.0}$	$6.6\ ^{+3.8}_{-2.7}$
λ_π	0.66 ± 0.15	0.66 ± 0.11

Because proton one-particle data are contaminated by the presence of spectator protons, we have not included them in our analysis thus far. However, because they are frequently

included in other analyses [14–18], we now consider the effect of including 331 data points for the proton one-particle multiplicity distribution corresponding to the central 7% of collisions in the same reaction that we have been considering [10]. A systematic error of 15% is used also for the proton one-particle multiplicity distribution. With a total of 1747 data points for the seven types of data considered and nine adjustable parameters, the number of degrees of freedom ν is 1738 in this case. The resulting value of χ^2 is 2546.6, which corresponds to an unacceptably large value of $\chi^2/\nu = 1.465$. The probability that a perfect model would have resulted in a value of χ^2 at least as large as that found here is the incredibly small value 1.1×10^{-33}. Nevertheless, for completeness, we give in the third column of Table V the values of nine independent freeze-out properties determined this way, along with their uncertainties at 99% confidence limits.

IV. FLAWS IN PREVIOUS ANALYSES

The results of the above detailed analyses indicate that the freeze-out temperature is less than 100 MeV and that both the longitudinal and transverse collective velocities—which are anti-correlated with the temperature—are substantial. Similar conclusions concerning a low freeze-out temperature have also been reached in Refs. [12,13]. However, other analyses [14–18] have yielded a much higher freeze-out temperature of approximately 140 MeV for both this reaction and other reactions involving heavier projectiles and/or higher bombarding energies. In order to reconcile this serious discrepancy, we now examine the features in these analyses that erroneously led them to the conclusion of a much higher freeze-out temperature. These analyses fall into two major classes, which we consider in turn.

A. Neglect of relativity in extrapolation of slope parameters to zero particle mass

One type of analysis [14,15] was based upon the extrapolation to zero particle mass of extracted slope parameters characterizing the dependence of unnormalized transverse one-particle multiplicity distributions upon transverse mass. For a given reaction and type of particle, this transverse one-particle multiplicity distribution was represented by the expression[2]

$$\frac{1}{m_{\mathrm{t}}}\frac{dN}{dm_{\mathrm{t}}} = A\exp\left(-\frac{m_{\mathrm{t}}}{T_{\mathrm{eff}}}\right) , \tag{8}$$

where A is an arbitrary normalization constant and T_{eff} is the extracted slope parameter. Values of T_{eff} were extracted in this way for six types of particles originating from three separate reactions, namely π^+, π^-, K^+, K^-, p, and $\bar{p}$ originating from the reaction $p + p$ at center-of-mass energy $\sqrt{s} = 23$ GeV, from the 10% most central collisions in the reaction

[2]To facilitate comparisons with our own expressions, we have transformed the notation used in Refs. [14,15] to that used here.

S + S at $p_{\text{lab}}/A = 200$ GeV/c, and from the 6.4% most central collisions in the reaction Pb + Pb at $p_{\text{lab}}/A = 158$ GeV/c.

As we will see below, the values of these extracted slope parameters contain valuable information, but they were unfortunately analyzed in Refs. [14,15] in terms of the erroneous equation

$$T_{\text{eff}} = T + m\bar{v}^2 \ , \tag{9}$$

where T is the nuclear temperature (whose value we are trying to determine) and $\bar{v}$ is the average transverse collective velocity of the expanding matter from which the particle originated. Alas, this equation neglects relativity—even though these are relativistic collisions! It was introduced on page 182c of Ref. [14] with the phrase "One may empirically guess a relationship between the slope parameter and particle mass," whereas the words describing the same equation on page 2082 of Ref. [15] are "The correlation between the slope parameter and particle mass m may be described qualitatively by the relationship ... " On the basis of the erroneous Eq. (9), the extrapolation in Ref. [15] of the extracted slope parameters to zero particle mass yielded the result $T \approx 140 \pm 15$ MeV.

In the limit in which the particle velocity is large compared to the average collective velocity and with the aid of other simplifying assumptions and approximations,[3] the correct relationship between slope parameter, nuclear temperature, particle mass, and average collective velocity can be easily derived from the relativistically correct Eq. (3). With the neglect of contributions from resonance decays, the neglect of the ∓ 1 appearing in the denominator of Eq. (3), the assumption of a constant freeze-out temperature, and the assumption that freeze-out occurs at a constant time t in the source frame, Eq. (3) leads to

$$\frac{1}{m_{\text{t}}}\,\frac{d^2N}{dy\,dm_{\text{t}}} = A'E\int_V d^3x \exp\left[-\frac{p\cdot v(\mathbf{x})}{T}\right] = A'E\int_V d^3x \exp\left\{-\frac{\gamma(\mathbf{x})[E - \mathbf{p}\cdot\mathbf{v}(\mathbf{x})]}{T}\right\} \ , \tag{10}$$

where A' is a different arbitrary normalization constant from the one appearing in Eq. (8), the subscript V on the integral denotes the spatial integration limits for the source, and the position-dependent Lorentz factor $\gamma(\mathbf{x}) = 1/\sqrt{1 - \mathbf{v}(\mathbf{x})\cdot\mathbf{v}(\mathbf{x})}$.

By introducing an average collective velocity $\bar{v}$ in the integrations in Eq. (10), taking the limit in which the particle velocity is large compared to the collective velocity, specializing to the transverse direction, and neglecting the pre-exponential E dependence, we are led to

$$\frac{1}{m_{\text{t}}}\,\frac{dN}{dm_{\text{t}}} = A\exp\left[-\frac{\bar{\gamma}(m_{\text{t}} - p_{\text{t}}\bar{v})}{T}\right] = A\exp\left(-\frac{m_{\text{t}} - \bar{v}\sqrt{m_{\text{t}}{}^2 - m^2}}{T\sqrt{1-\bar{v}^2}}\right) \ . \tag{11}$$

To obtain the relationship between the slope parameter, nuclear temperature, particle mass, and average collective velocity, we equate the derivatives with respect to m_{t} of Eqs. (8) and (11), which leads to

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[3]The expanding source model developed in Refs. [8,9] and used here does *not* require that the particle velocity be large compared to the average collective velocity and does *not* utilize these other simplifying assumptions and approximations.

$$T = \left(1 - \frac{\bar{v} m_{\rm t}}{p_{\rm t}}\right) \frac{T_{\rm eff}}{\sqrt{1-\bar{v}^2}} = \left(1 - \bar{v}\sqrt{1+\frac{m^2}{{p_{\rm t}}^2}}\right) \frac{T_{\rm eff}}{\sqrt{1-\bar{v}^2}} \; . \tag{12}$$

An analogous relationship has also been obtained by Siemens and Rasmussen [28] for the case of a blast wave produced by the explosion of a spherically symmetric fireball.

In the limit of zero particle mass, Eq. (12) reduces to

$$T = T_{\rm eff}\sqrt{\frac{1-\bar{v}}{1+\bar{v}}} \; , \tag{13}$$

which agrees with the result obtained by Schnedermann, Sollfrank, and Heinz [29,30] for the case of cylindrical symmetry. With a typical value of 0.4 c for the average collective velocity $\bar{v}$ and the limiting value of $T_{\rm eff} \approx 140 \pm 15$ MeV obtained in Ref. [15] by extrapolating slope parameters to zero particle mass, Eq. (13) yields $T \approx 92 \pm 10$ MeV for the nuclear temperature.

B. Accumulation of effects from several approximations made in a thermal model

Another type of analysis [14–18] utilized the thermal model of Schnedermann, Sollfrank, and Heinz [29,30] to extract the nuclear temperature and transverse surface collective velocity from unnormalized experimental transverse one-particle multiplicity distributions. An accumulation of effects from several approximations led to a somewhat higher temperature than we have found in our expanding source model [8,9]. These approximations include the neglect of contributions from resonance decays, the neglect of the ∓ 1 appearing in the denominator of Eq. (3), the neglect of the coupling of the transverse motion to the longitudinal motion, and—most importantly—the neglect of information contained in the absolute normalization of the multiplicity distributions. The accumulation of effects from these approximations was responsible for the conclusion on page 2083 of Ref. [15] that "Within a temperature range $100 \leq T \leq 150$ MeV, the fits are equally good." Clearly, the use of unnormalized experimental transverse one-particle multiplicity distributions in such a thermal model cannot be expected to provide a definitive determination of the nuclear temperature at freeze-out.

V. SUMMARY AND CONCLUSION

We have used a nine-parameter expanding source model that includes special relativity, quantum statistics, resonance decays, and freeze-out on a realistic hypersurface in space-time to analyze in detail invariant π^+, π^-, K^+, and K^- one-particle multiplicity distributions and π^+ and K^+ two-particle correlations in nearly central collisions of Si + Au at $p_{\rm lab}/A = 14.6$ GeV/c. By considering separately the one-particle data and the correlation data, we found that the central baryon density, nuclear temperature, transverse collective velocity, longitudinal collective velocity, and source velocity are determined primarily by one-particle multiplicity distributions and that the transverse radius, longitudinal proper time, width in proper time, and pion incoherence fraction are determined primarily by two-particle correlations. By considering separately the pion data and the kaon data, we found

that although the pion freeze-out occurs somewhat later than the kaon freeze-out, the 99% confidence-level error bars associated with the two freeze-outs overlap.

By constraining the transverse freeze-out to the same source time for all points with the same longitudinal position and by allowing a more flexible freeze-out in the longitudinal direction, we found that the precise shape of the freeze-out hypersurface is relatively unimportant. By regarding the pion and kaon one-particle data to be unnormalized, we found that the nuclear temperature increases slightly, but that its uncertainty increases substantially. By including proton one-particle data (which are contaminated by spectator protons), we found that the nuclear temperature increases slightly. These detailed studies confirm our earlier conclusion [8,9] based on the simultaneous consideration of the pion and kaon one-particle and correlation data that the freeze-out temperature is less than 100 MeV and that both the longitudinal and transverse collective velocities—which are anti-correlated with the temperature—are substantial.

We also discussed the flaws in previous analyses that yielded a much higher freeze-out temperature of approximately 140 MeV for both this reaction and other reactions involving heavier projectiles and/or higher bombarding energies. One type of analysis was based upon the use of an erroneous equation that neglects relativity to extrapolate slope parameters to zero particle mass. Another type of analysis utilized a thermal model in which there was an accumulation of effects from several approximations.

The future should witness the arrival of much new data on invariant one-particle multiplicity distributions and two-particle correlations as functions of bombarding energy and/or size of the colliding nuclei. The proper analysis of these data in terms of a realistic model could yield accurate values for the density, temperature, collective velocity, size, and other properties of the expanding matter as it freezes out into a collection of noninteracting hadrons. A sharp discontinuity in the value of one or more of these properties could conceivably be the long-awaited signal for the formation of a quark-gluon plasma or other new physics.

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